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General Certificate of Education (A-level)
June 2012

Mathematics MPC4

(Specification 6360)

Pure Core 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
$\surd$or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q	Solution	Marks	Total	Comments
1(a)(i)	$5x-6=A(x-3)+Bx$ $x=0 \quad x=3$	M1		Multiply by denominator and use two values of *x*.
	$A=2 \quad B=3$	A1	2	
	Alternative: equate coefficients $-6=-3A \quad 5=A+B$	(M1)		Set up and solve simultaneous equations for values of *A* and *B*.
	$A=2 \quad B=3$	(A1)		
(ii)	$\left(\int \frac{2}{x}+\frac{3}{x-3}\,dx=\right) 2\ln x$	B1ft		their *A* ln *x*
	$+3\ln(x-3) \quad (+C)$	B1ft	2	their *B* ln (*x* – 3) and no other terms; condone *B* ln *x* – 3
(b)(i)	$2x^2-x+3$ $2x+1\overline{)4x^3+5x-2}$ $4x^3+2x^2$ $-2x^2+5x$ $-2x^2-x$ $6x-2$ $6x+3$ -5	M1		Division as far as $2x^2+px+q$ with $p\neq 0,\ q\neq 0$, PI
	$p=-1$	A1		PI by $2x^2-x+q$ seen
	$q=3$	A1		PI by $2x^2-x+3$ seen
	$r=-5$	A1	4	and must state *p*= –1, *q* = 3, *r* = –5 explicitly or write out full correct RHS expression
	Alternative 1: $4x^3+5x-2=$ $4x^3+(2+2p)x^2+(p+2q)x+q+r$ $2+2p=0$ $p+2q=5$ $q+r=-2$	(M1)		Clear attempt to equate coefficients, PI by *p* = –1
	$p=-1$	(A1)		
	$q=3 \quad r=-5$	(A1A1)		
	Alternative 2: $4x^3+5x-2=(2x+1)(2x^2+px+q)+r$ $x=-\frac{1}{2} \quad 4\times\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)+2=r$	(M1)		$x=-\frac{1}{2}$ used to find a value for r
	$r=-5$	(A1)		
	$p=-1\ ,\ q=3$	(A1A1)		

MPC4

Q	Solution	Marks	Total	Comments
(b)(ii)	$\left(\dfrac{4x^3+5x-2}{2x+1}=\right)2x^2+px+q+\dfrac{r}{2x+1}$	M1		
	$\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+3x+k\ln(2x+1)\quad(+C)$	A1ft		ft on p and q
	$\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+3x-\dfrac{5}{2}\ln(2x+1)\quad(+C)$	A1	3	CSO
	Total		**11**	
2(a)	$R=\sqrt{10}$	B1		Accept 3.2 or better. Can be earned in (b)
	$\tan\alpha=3$	M1		OE; M0 if $\tan\alpha=-3$ seen
	$\alpha=71.6$ or better	A1	3	$\alpha=71.56505...$
(b)	$\sin(x\pm\alpha)=\dfrac{-2}{R}$	M1		or their R and/or their α; PI
	$x(=-39.2+71.6)\quad=32(.333)$	A1		32 or better Condone 32.4
	or			
	$x-71.6=219.2$	m1		must see 219 and 72 or better PI by 291 or better as answer Condone extra solutions
	$x=\ 291$	A1	4	Condone 290.8 or better CSO Withhold final A1 if more than two answers given within interval
	Total		**7**	

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Q	Solution	Marks	Total	Comments
3(a)	$(1+4x)^{\frac{1}{2}} = 1+4\times\frac{1}{2}x+kx^2$	M1		
	$= 1+2x-2x^2$	A1	2	
(b)(i)	$(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$	B1		OE $\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} =$			
	$1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^2$	M1		Condone missing brackets and use of $\left(+\frac{x}{4}\right)$ instead of $\left(-\frac{x}{4}\right)$
	$=1+\frac{1}{8}x+\frac{3}{128}x^2$			
	$(4-x)^{-\frac{1}{2}} = \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	3	CSO $0.5+0.0625x+0.0117(1875)x^2$
	Alternative using formula from FB			
	$(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)\times 4^{-\frac{3}{2}}(-x)$	(M1)		Condone one error and missing brackets
	$+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\times 4^{-\frac{5}{2}}(-x)^2$			
	$=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	(A2)		CSO Must be fully correct
(b)(ii)	$-4<x<4$ or $x<4$ and $x>-4$	B1	1	Condone $\lvert x\rvert<4$ Must be **and**; not **or** not , (comma)
(c)	$\sqrt{\frac{1+4x}{4-x}} = (1+4x)^{\frac{1}{2}}(4-x)^{-\frac{1}{2}}$			
	$=\left(1+2x-2x^2\right)\left(\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2\right)$	M1		product of their expansions
	$=\frac{1}{2}+\frac{17}{16}x-\frac{221}{256}x^2$	A1	2	CSO $0.5+1.0625x-0.8632(8...)x^2$
	Total		**8**	

MPC4

Q	Solution	Marks	Total	Comments
4(a)(i)	$1000\times1.03^5 \approx (\text{£})1160$	B1	1	Condone missing £ sign;1160 only.
(ii)	$2000 < 1000\left(1+\dfrac{3}{100}\right)^n$	B1		Condone '=' or '<' used throughout
	$\ln 2 < n\ln 1.03$	M1		Take logs, any base, of their initial expression **correctly**
	$(n > 23.449....)\quad (N=)24$	A1	3	Condone 23
(b)	$1000\times\left(1+\dfrac{3}{100}\right)^n > 1500\times\left(1+\dfrac{1.5}{100}\right)^n$	B1		Condone use of T for n Condone '=' or '<' used throughout
	$\ln 1000 + n\ln 1.03 > \ln 1500 + n\ln 1.015$	M1		Take logs, any base, of their initial expression **correctly**
	$n > \dfrac{\ln(1.5)}{\ln\left(\dfrac{1.03}{1.015}\right)}$	A1		Correct expression for n or T
	$(n > 27.63...)\quad (T=)28$	A1	4	Condone 27
	Total		**8**	

MPC4

Q	Solution	Marks	Total	Comments
5 (a)(i)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{6\cos 2\theta}{-2\sin\theta}$	M1 A1		condone coefficient errors
	$= \frac{6(1-2\sin^2\theta)}{-2\sin\theta}$	m1		Use $\cos 2\theta = 1 - 2\sin^2\theta$
	$= 6\sin\theta - 3\operatorname{cosec}\theta$	A1	4	$a = 6 \quad b = -3$
(a)(ii)	$\theta = \frac{\pi}{6} \quad \frac{dy}{dx} = 6\times\frac{1}{2} - 3\times 2 = -3$	B1ft		$\theta = \frac{\pi}{6}$ substituted into their $\frac{dy}{dx}$ and evaluated
	gradient normal $= \frac{1}{3}$	B1ft	2	ft $\frac{dy}{dx}$, provided non-zero
(b)	$y = 6\sin\theta\cos\theta$ $= (\pm)6\sqrt{1-\cos^2\theta}\times\cos\theta$	M1		Correct expansion of $\sin 2\theta$ and use $x = 2\cos\theta$ to eliminate θ
	$= (\pm)6\sqrt{1-\left(\frac{x}{2}\right)^2}\times\left(\frac{x}{2}\right)$	A1		Correct elimination of θ
	$y^2 = \frac{9}{4}x^2(4-x^2)$	A1	3	$p = \frac{9}{4}$ OE and $(4-x^2)$ shown
	Alternative using verification $y^2 = 9\sin^2 2\theta = 36\sin^2\theta\cos^2\theta$	(M1)		must be squared
	$x^2(4-x^2) = 4\cos^2\theta\times 4\sin^2\theta$	(A1)		
	$p = \frac{9}{4}$ OE	(A1)		or $y^2 = \frac{9}{4}x^2(4-x^2)$
	Total		**9**	

MPC4

Q	Solution	Marks	Total	Comments
6	$9x^2-6xy+4y^2 \quad =3$			
	$18x \quad =0$	B1		=0 PI
	$-6y-6x\dfrac{dy}{dx}$	B1		or $\dfrac{d(6xy)}{dx}=6y+6x\dfrac{dy}{dx}$ seen separately
	$+8y\dfrac{dy}{dx}$	B1		$\dfrac{dy}{dx}(-6x+8y)=6y-18x$
	Use $\dfrac{dy}{dx}=0$	M1		
	$\Rightarrow y=3x$ or $x=\dfrac{y}{3}$	A1		CSO
	$y=3x\Rightarrow 9x^2-6x\times 3x+4(3x)^2=3$	m1		Substitute $y=ax$ into equation and solve for a value of x or y. Condone missing brackets.
	$27x^2=3\Rightarrow x=\pm\dfrac{1}{3}$ OE	A1ft		Both values of x or y required. ft on their $y = 3x$
	$\left(\dfrac{1}{3},1\right)$ $\left(-\dfrac{1}{3},-1\right)$	A1	8	CSO Correct corresponding simplified values of x and y. Withhold if additional answers given
	Total		**8**	

MPC4

Q	Solution	Marks	Total	Comments
7(a)	$2\lambda = 8 + 2\mu$ $-2 \quad = 3 + 5\mu$ $\lambda = 3\ ,\ \mu = -1$	M1		Use the first two equations to set up and attempt to solve simultaneous equations for λ or μ. Must not assume $q = 4$.
	$q - \lambda = 5 + 4\mu$ $q = 5 + 3 - 4 = 4$	A1		Use 3rd equation to show $q = 4$ **AG**.
	P is at $(6, -2, 1)$	B1	3	Condone as a column vector
(b)	$\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} = 4 - 4 = 0 \Rightarrow$ perpendicular	B1	1	or $2\times2 + -1\times4 = 0$ seen **and** conclusion (condone $\theta = 90$)
(c)(i)	A is at (2, –2, 3) $AP^2 = (6-2)^2 + (-2--2)^2 + (1-3)^2$	M1		Candidate's $\lvert\overrightarrow{AP}\rvert^2$
	$= 20$	A1	2	CAO NMS $AP = \sqrt{20}$ M1A0
(ii)	$\left(\overrightarrow{PB} =\right) \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix} \quad \left(= \begin{bmatrix} 2+2\mu \\ 5+5\mu \\ 4+4\mu \end{bmatrix} \right)$	M1		Clear attempt to find $\overrightarrow{BP}$ or $\overrightarrow{PB}$ in terms of μ
	$\left(PB^2 =\right)(2+2\mu)^2 + (5+5\mu)^2 + (4+4\mu)^2$	m1		Find distance BP in terms of μ
	$45\mu^2 + 90\mu + 45 = 20$ $(5)(9\mu^2 + 18\mu + 5) = 0$	m1		Attempt to set up three-term quadratic in μ and equate to their AP^2
	$(3\mu + 1)(3\mu + 5) = 0$	m1		Solve quadratic equation to obtain **two** values of μ
	$\mu = -\frac{1}{3}$ and $\mu = -\frac{5}{3}$	A1		Both values correct.
	B is at $\left(\frac{22}{3}, \frac{4}{3}, \frac{11}{3}\right)$ and $\left(\frac{14}{3}, -\frac{16}{3}, -\frac{5}{3}\right)$	A1	6	Both sets of coordinates required. Condone column vectors. SC one value of μ correct and corresponding coordinates of B correct scores A1 A0.

MPC4

Q	Solution	Marks	Total	Comments
	Alternative 1			
	$\left(\overrightarrow{AB}=\right)\begin{bmatrix}8\\3\\5\end{bmatrix}+\mu\begin{bmatrix}2\\5\\4\end{bmatrix}-\begin{bmatrix}2\\-2\\3\end{bmatrix}\quad\left(=\begin{bmatrix}6+2\mu\\5+5\mu\\2+4\mu\end{bmatrix}\right)$	(M1)		Clear attempt to find $\overrightarrow{AB}$ or $\overrightarrow{BA}$ in terms of μ
	$\left(AB^2=\right)(6+2\mu)^2+(5+5\mu)^2+(2+4\mu)^2$	(m1)		Find distance AB in terms of μ
	$45\mu^2+90\mu+65=40$ $(5)\left(9\mu^2+18\mu+5\right)=0$	(m1)		Attempt to set up three-term quadratic in μ and equate to their $2\times$ their AP^2
	As before			
	Alternative 2			
	$\overrightarrow{PB}=k\begin{bmatrix}2\\5\\4\end{bmatrix}$	(M1)		
	$k^2\left(2^2+5^2+4^2\right)=20$	(m1) (m1)		m1 for LHS m1 for equating to ‘their 20’ May score M1m0m1
	$k=\pm\frac{2}{3}$	(A1)		
	$\overrightarrow{OB}=\overrightarrow{OP}+(\pm)(\text{their value of }k)\begin{bmatrix}2\\5\\4\end{bmatrix}$	(m1)		
	B is at $\left(\frac{22}{3},\frac{4}{3},\frac{11}{3}\right)$ and $\left(\frac{14}{3},-\frac{16}{3},-\frac{5}{3}\right)$	(A1)		
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	$\frac{\mathrm{d}h}{\mathrm{d}t}$	B1		
	$derivative = * \times (2-h)$	M1		Use of $2-h$ or $h-2$; $*$ is a constant or expression in h and/or t.
	$\frac{\mathrm{d}h}{\mathrm{d}t} = k(2-h)$	A1	3	All correct; must be $(2-h)$
(b)(i)	$\int x\sqrt{2x-1}\ \mathrm{d}x = \int \frac{1}{15}\mathrm{d}t$	B1		Correct separation and notation; condone missing integral signs.
	$= \frac{1}{15}t$	B1		
	Substitute $u = 2x-1$ $\int x\sqrt{2x-1}\ \mathrm{d}x = \int \frac{1}{2}(u+1)\sqrt{u}\ \frac{1}{2}\mathrm{d}u$	M1		Suitable substitution and attempt to write integral in terms of u including dx replaced by $\frac{1}{2}\mathrm{d}u$ or 2 du.
	$= \left(\frac{1}{4}\right)\int u^{\frac{3}{2}} + u^{\frac{1}{2}}\ \mathrm{d}u$	A1		$\frac{1}{4}$ need not be seen
	$= \frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right)\ (+C)$	A1		Integration correct including $\frac{1}{4}$
	$x=1$, $t=0$ $u=1, t=0\quad \frac{1}{4}\left(\frac{2}{5}+\frac{2}{3}\right) + C = 0$	M1		Use $x=1$, $t=0$ to find a value for constant C from equation in x and t.
	$C = -\frac{4}{15}$	A1		$C = -0.2666...$ $C = -0.267$ or better
	$t = \frac{1}{2}\left(3(2x-1)^{\frac{5}{2}} + 5(2x-1)^{\frac{3}{2}}\right) - 4$	A1	8	ISW $t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$
	Alternative (Parts) As before	(B1B1)		
	$u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = (2x-1)^{\frac{1}{2}}$	(M1)		Attempt to use parts
	$\mathrm{d}u = 1\quad v = k(2x-1)^{\frac{3}{2}}$			
	$\int x\sqrt{2x-1}\ \mathrm{d}x = x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \int \frac{1}{3}(2x-1)^{\frac{3}{2}}\ \mathrm{d}x$	(A1)		Condone missing dx
	$= x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}}\ (+C)$	(A1)		
	$x=1$, $t=0\quad \frac{1}{3} - \frac{1}{15} + C = 0$	(M1)		Use $x=1$, $t=0$ to find a value for constant C from equation in x and t
	$C = -\frac{4}{15}$	(A1)		$C = -0.2666...$ $C = -0.267$ or better
	$t = 5x(2x-1)^{\frac{3}{2}} - (2x-1)^{\frac{5}{2}} - 4$	(A1)		ISW $t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$
(ii)	$x = 2\quad t = 32.4\quad$ (minutes)	B1	1	32.4 or better (32.373…)
	Total		**12**	
	TOTAL		**75**	